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Formula Relating Fracture Strength and Fracture Ductility with Strength Coefficient and Strain-Hardening Exponent

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To theoretically calculate the strength coefficient and the strain-hardening exponent with conventional mechanical property parameters, formulas relating them with fracture strength and fracture ductility are studied using test data for ten alloys. The applicability of the traditional formula relating these four material constants is discussed first, and then new formulas are proposed based on the premise that the traditional approach cannot be used. The main conclusions made herein are that only under certain conditions can the traditional formula be used to describe the relationship among fracture strength, fracture ductility, strength coefficient, and strain-hardening exponent; otherwise, a new formula must be used.

Keywords fracture ductility, fracture strength, Hollomon equation, strain-hardening exponent, strength coefficient

1. Introduction

When metal fatigue crack initiation life is predicted using the equivalent stress amplitude method (Ref 1-3), or when metal tensile properties are studied, the strength coefficient and the strain-hardening exponent of the metal must be known. Although the two material constants can be determined experimentally, they are often calculated theoretically because calculating them experimentally is expensive and time-consuming.

Traditionally, two formulas based on the Hollomon equation have been used to theoretically calculate the strength coefficient and the strain-hardening exponent. One equation relates the strength coefficient and the strain-hardening exponent with the yield strength and the yield strain (Ref 4), while the other relates them with the fracture strength and the fracture ductility (Ref 4-6). Reference 7 examines the applicability of the first formula. It was found that the formula does not properly describe the relationship among yield strength, yield strain, strength coefficient, and strain-hardening exponent. Therefore, new formulas are proposed. Similarly, in the following study the second formula does not precisely express the relationship among fracture strength, fracture ductility, strength coefficient, and strain-hardening exponent for all alloys. Therefore, it was necessary to find a formula for the alloys that more accurately describes their behavior. Only when alloy behavior is more accurately described can metal fatigue crack initiation life be predicted by the equivalent stress amplitude method.

Thus, the purpose of the research was to define a relationship among fracture strength, fracture ductility, strength coefficient, and strain-hardening exponent where the traditional formula does not hold well.

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2. Traditional Formula Relating Four Material Constants

The Hollomon equation is a basic equation correlating the stress to the strain (Ref 8):

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n} \quad \text{(Eq 1)}$$

$$\sigma = K\varepsilon_p^n \quad \text{(Eq 2)}$$

Nomenclature

E	Young’s modulus
K	strength coefficient
n	strain-hardening exponent
ε	total strain
ε_f	fracture ductility
ε_p	plastic strain
σ	stress
σ_b	ultimate tensile strength
σ_f	fracture strength
$\sigma_{0.2}$	yield strength
σ_{f1}	theoretical fracture strength
σ_{f2}	theoretical fracture strength
α	new fracture ductility parameter
ψ	reduction of area

In Eq 1 and 2, σ is the stress, ε is the total strain, ε_p is the plastic strain, E is the Young's modulus, K is the strength coefficient, and n is the strain-hardening exponent.

If Eq 2 is used to correlate fracture strength σ_f with the fracture ductility ε_f, and for the case when fracture ductility ε_f is much greater than the elastic strain ε_e (i.e., $\varepsilon_f \gg \varepsilon_e$), then:

$$\sigma_f = K\varepsilon_p^n \qquad \text{(Eq 3)}$$

Equation 3 is defined as the "traditional" formula. It is used on the one hand to calculate the strength coefficient and the strain-hardening exponent (Ref 4-6) and on the other hand to predict metal fatigue crack initiation life using the equivalent stress amplitude method (Ref 1-4). In so far as the first application is concerned, it will be shown that the traditional formula does not precisely express the relationship among fracture strength, fracture ductility, strength coefficient, and strain-hardening exponent for all alloys. For the second application, it has been shown for some alloys that fatigue crack initiation life predicted by the equivalent stress amplitude method correlates well with experimental data, while for other alloys, fatigue crack initiation life deviates significantly from the test data (Ref 1-6). There are many reasons for this behavior, but the intrinsic limitation of Eq 3 may be an important factor.

Equations 1 and 2 are fitted relations using tensile test data (i.e., σ and ε). In Ref 7, it was shown that this approach did not adequately express the stress-strain relation at the yield point for all alloys. Similarly, if Eq 3 is deduced from Eq 2, a check should be made to see if it properly expresses the relationship between fracture strength and fracture ductility.

In Ref 9-11, the performance parameters from the experiment on ten alloys were provided, including fracture strength, fracture ductility, strength coefficient, strain-hardening exponent, etc. Using these parameters in Eq 3, i.e., fracture ductility, strength coefficient, and strain-hardening exponent, the fracture ductility can be calculated. If the fracture ductility values, given in Ref 9-11, are assumed to be true values, then the calculated results using Eq 3 are assumed to be the theoretical ones. Comparing the results allows the "correctness" and "precision" of Eq 3 to be deduced through comparison of the theoretical fracture strength with the (assumed) true one.

The performance parameters of ten alloys (Ref 9-11) and the theoretical calculated fracture strengths are listed in Table 1 and 2. In these tables, ψ is the percentage reduction in area, $\sigma_{0.2}$ is the yield strength, σ_b is the ultimate tensile strength, σ_f is the fracture strength, ε_f is the fracture ductility, and σ_{f1} is the theoretical fracture strength ($\delta\sigma_{f1} = (\sigma_{f1} - \sigma_f)/\sigma_f$). σ_f, σ_{f1}, $\sigma_{0.2}$, σ_b, and K are in units of MPa.

It can be seen from Table 1 that for LC4CS, the theoretical fracture strength (σ_{f1}) is smaller than the listed value, while for the other three alloys, σ_{f1} is greater than the true values. The maximum deviation between them is only 7.6%, which implies that the theoretical fracture strengths are approximately equal to the true ones. Therefore, for the four alloys listed in Table 1, Eq 3, or the traditional formula, is suitable for expressing the relationship among fracture strength, fracture ductility, strength coefficient, and strain-hardening exponent. However, in Table 2, not only is σ_{f1} smaller than σ_f, but the minimum deviation between them is –5.7%. In other words, for the six alloys listed in Table 2, Eq 3 does not accurately express the relationship among fracture strength, fracture ductility, strength coefficient, and strain-hardening exponent. Therefore, a new formula relating the four material constants should be determined.

If the data in Table 1 are studied carefully, it is noted that:

$$\psi\varepsilon_f < 5\% \qquad \text{(Eq 4)}$$

or

$$10\% < \psi\varepsilon_f < 20\% \qquad \text{(Eq 5)}$$

Table 1 Alloy parameters and theoretical fracture strengths

Material	LY12CZ, rod	LC4CS	2024-T4	7075-T6
ψ, %	16.5	16.6	35.0	33.0
ε_f, %	18	18	43	41
σ_f	643	711	634	745
K	850	775	807	827
n	0.158	0.063	0.200	0.113
$\psi\varepsilon_f$, %	2.97	2.99	15.05	13.53
σ_{f1}	648	696	682	748
$\delta\sigma_{f1}$, %	0.7	–2.1	7.6	0.4
σ_b	545	614	476	579
$\sigma_{0.2}$	400	571	303	469
$\sigma_b/\sigma_{0.2}$	1.36	1.08	1.57	1.23
H/*S*	*H*	*H*	*H*	*H*

Source: Ref 9-11

Table 2 Alloy parameter and theoretical fracture strengths

Material	LY12CZ, plate	LC9CGS3	30CrMnSiA	30CrMnSiNi2A	40CrMnSiMoVA	AISI 4340
ψ, %	26.6	21.0	53.6	52.3	43.7	57
ε_f, %	30	28	77	74	63	84
σ_f	618	748	1795	2601	3512	1655
K	545	725	1476	2356	3150	1579
n	0.089	0.071	0.063	0.091	0.147	0.066
σ_b	476	560	1177	1655	1875	1241
$\sigma_{0.2}$	331	518	1105	1308	1513	1179
$\psi\varepsilon_f$, %	8.03	5.95	41.41	38.70	27.67	47.9
σ_{f1}	490	663	1452	2292	2946	1561
$\delta\sigma_{f1}$, %	–20.7	–11.5	–19.1	–11.9	–16.1	–5.7
σ_{f2}	703	716	1552	2900	3651	1643
$\delta\sigma_{f2}$, %	13.8	–4.3	–13.6	11.5	4.0	–0.7
$\sigma_b/\sigma_{0.2}$	1.43	1.08	1.07	1.27	1.24	1.05
H/*S*	*H*	*H*	*S*	*S*	*S*	*S*

Source: Ref 9-11

or

$$K > \sigma_f \quad \text{(Eq 6)}$$

As discussed above, the traditional formula can be used to describe the relationship among fracture strength, fracture ductility, strength coefficient, and strain-hardening exponent, so relations 4-6 express the applicability of the traditional formula.

In Ref 7, a new fracture ductility parameter α has been introduced and is defined as:

$$\alpha = \psi\varepsilon_f = -\psi \ln(1 - \psi) \quad \text{(Eq 7)}$$

Because ψ, a percentage reduction in area, reflects the fracture ductility of material, α also reflects the fracture ductility of material. In Ref 7, α has been used to describe the applicability of the formulas, correlating the strength coefficient and the strain-hardening exponent with yield strength and yield strain. Now, if α is also used to describe the applicability of the traditional formula for the alloys listed in Table 1, then inequalities 4 and 5 become:

$$\alpha < 5\% \quad \text{(Eq 8)}$$

or

$$10\% < \alpha < 20\% \quad \text{(Eq 9)}$$

In addition, the new fracture ductility parameter, α, may be a better parameter at describing the hardening behavior of alloys than the ratio of σ_b to $\sigma_{0.2}$. In general, the metal hardening behavior (Ref 12, 13) is described by the value $\sigma_b/\sigma_{0.2}$. When $\sigma_b/\sigma_{0.2} > 1.4$, the alloy behaves in a cyclic hardening manner. When $\sigma_b/\sigma_{0.2} < 1.2$, the alloy behaves in a cyclic softening manner. For the case when $1.2 < \sigma_b/\sigma_{0.2} < 1.4$, alloy behavior is not well defined. However, in view of the new fracture ductility parameter, when $\alpha > 20\%$ the alloy behaves in a cyclic softening manner, but when $\alpha < 20\%$ the alloy behaves in a cyclic hardening manner. For completeness, the hardening behavior of the alloys is denoted as *H*/*S*, and these values are also listed in the two tables (Ref 7). In view of hardening/softening behavior, the alloys in Table 1 behave in a cyclic hardening manner, and some of the alloys in Table 2 behave in a cyclic hardening manner while others cyclic soften. Therefore, it seems that for cyclic softening alloys, the traditional formula can not be used to express the relationship among fracture strength, fracture ductility, strength coefficient, and strain-hardening exponent.

3. New Formula Relating Four Material Constants

As has been pointed out above, when σ_{f1} deduced from the traditional formula are smaller than σ_f, the traditional formula can not be used to express the relationship among material parameters. Considering the fact that σ_b is always greater than $\sigma_{0.2}$, and examining the magnitude of $\sigma_b/\sigma_{0.2}$, and the relative deviation $\delta\sigma_{f1}$,

$$|\delta\sigma_{f1}| \approx \frac{\sigma_b}{\sigma_{0.2}} - 1 \quad \text{(Eq 10)}$$

Thus, the factor $\sigma_b/\sigma_{0.2}$ seems appropriate for "correcting" Eq 3 for small deviations. Therefore,

$$\sigma_f = \frac{\sigma_b}{\sigma_{0.2}} K\varepsilon_f^n \quad \text{(Eq 11)}$$

To evaluate the "correctness" and "precision" of Eq 11, six material constants (i.e., strength coefficient, strain-hardening exponent, yield strength, ultimate tensile strength, fracture strength, and fracture ductility) of the alloys are taken as true values, while the calculated result from Eq 11 is taken as the theoretical value and is denoted as σ_{f2}. By comparing the theoretical fracture strength with the true one, and by defining $\delta\sigma_{f2} = (\sigma_{f2} - \sigma_f)/\sigma_f$, it can be seen that relative to σ_{f1} (Eq 3), σ_{f2} (Eq 11) fits the experimental data better.

When comparing Eq 11 with Eq 3, the factor $\sigma_b/\sigma_{0.2}$ appears. The theoretical fracture strengths derived from the traditional formula are all smaller than the true ones, with the minimum deviation between them equal to 5.7%. When the $\sigma_b/\sigma_{0.2}$ factor is used, not all the theoretical fracture strengths from Eq 11 are smaller than the true ones, with the maximum deviation between them equal to 13.8%. Therefore, Eq 11 more accurately represents the relationship among fracture strength, fracture ductility, strength coefficient, and strain-hardening exponent for the alloys listed in Table 2.

Furthermore, the applicability of the new Eq 11 can be obtained from Table 2, and from inequalities (Eq 4-6), that is:

$$5\% < \psi\varepsilon_f < 10\% \quad \text{(Eq 12)}$$

or

$$\psi\varepsilon_f > 20\% \quad \text{(Eq 13)}$$

or

$$K < \sigma_f \quad \text{(Eq 14)}$$

Again, if the applicability of the new formula (Eq 11) is described by α, then Eq 12 and 13 change to:

$$5\% < \alpha < 10\% \quad \text{(Eq 15)}$$

or

$$\alpha > 20\% \quad \text{(Eq 16)}$$

4. Conclusions

The purpose of the present paper was to evaluate formulas for expressing the relationship among fracture strength, fracture ductility, strength coefficient, and strain-hardening exponent. For this purpose, the applicability of the traditional formulae was studied, and the bounds on its use were determined.

- The applicability of the traditional formula occurs when $\alpha < 5\%$, or when $10\% < \alpha < 20\%$, or when $K > \sigma_f$. When any of the conditions are met, the traditional formula can be used to express the relationship among the tensile parameters, i.e.,

$$\sigma_f = K\varepsilon_f^n$$

- When the traditional formula does not hold, a new formula is proposed for conditions where $5\% < \alpha < 10\%$, or $\alpha > 20\%$, or $K < \sigma_f$.

$$\sigma_f = \frac{\sigma_b}{\sigma_{0.2}} K\varepsilon_f^n$$

- In view of hardening/softening behavior, it seems that for cyclic softening alloys, the traditional formula can not be universally used to express the relationship among fracture strength, fracture ductility, strength coefficient, and strain-hardening exponent.

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